MULTIVARIABLE STATE-SPACE IDENTIFICATION IN THE DELTA AND SHIFT OPERATORS

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ABSTRACT

This paper develops algorithms for multivariable state-space identification which can be used to estimate models in any operator of interest i.e., delta-rule, shift, Laplace s, etc. The approach is based on the State-Space from Frequency Data (SSFD) algorithm which was designed specifically to eliminate distortions from windowing effects. An important aspect of the approach is the use of overparametrization. A theoretical result is proved which demonstrates that the extra dynamics introduced from overparametrizing in the shift operator are stable, while the extra dynamics introduced from overparametrizing in the Laplace s and delta operators are generically unstable. This leads to certain modifications of the Laplace and delta operators to ensure stability under overparametrization. The usefulness of the identification algorithm is demonstrated on data taken from a 4-input/3-output flexible structure experiment, resulting in an identified state-space model with 100 states accurate over a 100 Hertz bandwidth.

1. INTRODUCTION

Recently, it has been found that realization algorithms based on Markov parameters [8][9] [10] can be effectively applied to the problem of state-space system identification. To date, these algorithms have been developed primarily in the time-domain [9][12] [14]. However, in many applications frequency rather than time domain data is available. In order to apply realization algorithms to this case, one must compute the Markov parameters from frequency data. It is at this point that windowing distortions are often introduced. For example, an Inverse Discrete Fourier Transform (IDFT) of the frequency data provides an estimate of the Markov parameter sequence which is distorted by time-aliasing effects [15]. The State-Space Frequency Domain (SSFD) algorithm was introduced in [6] to avoid such windowing distortions. The basic idea is to generate Markov parameters indirectly from a transfer function which has been curve fitted to the frequency data using the methods found in [4].

In this paper the SSFD algorithm is extended to estimate state-space models in any operator of interest, i.e., Laplace s, delta-rule, shift, etc.. from frequency domain data. In pursuing this extension, it is required to analyze the effect of overparametrization. It will be shown that the shift operator is particularly well suited for the job, i.e., the extra dynamics introduced by overparametrization will always be stable. In contrast, it will be shown that the delta operator and Laplace operator are poorly suited for the job i.e., the extraneous dynamics from overparametrizing are generically unstable. The main theoretical result of this paper characterizes the extraneous dynamics due to overparametrization, and is used to suggest appropriate modifications of the delt - rule and Laplace operators.

The SSFD algorithm is then demonstrated on multivariable data set from a 4-input/3-output flexible structure experiment. Complex curve fitting is performed based on the algorithms and sparse matrix methods given in [4], demonstrating the successful estim - tion of 780 parameters in the fitted transfer function. Markov parameter est imates are generated from the estimated transfer function according to the SSFD approach, leading to a (reduced) multivariable state- space model with 1 00 states accurate over a bandwidth 1 00 of Hertz.

2. STATE-SPACE FREQUENCY DOMAIN IDENTIFICATION

In this section, the SSFD algorithm of [6] is reviewed, and extended to the estimation of state- space models in the arbitrary operator ξ.

State-Space Frequency Domain (SSFD) Identification Algorithm

Step 1. *Curve fit the frequency response data $\mathcal{G}(\omega_i)$ $i = 1,...,m$, to find the transfer function G which minimizes or approximately minimizes the following 2-norm criteria,*

$$\min_{G} \sum_{i=1}^{m} w^2(\omega_i) \left\| \mathcal{G}(\omega_i) - G(\xi(\omega_i)) \right\|_f^2 \tag{1}$$

Here, the complex operator ξ is arbitrary; the data $\mathcal{G}(\omega_i)$, i = 1, ... m is given by noisy Values of the transfer function matrix evaluated over a grid Of m frequency points; $w(\omega_i)$ is a specified weighting function of frequency; and the Frobenious norm is defi ned as,

$$||X||_f^2 = Tr\{X^* X\} \tag{2}$$

with " $*$ " denoting the complex conjugate transpose. For optimization purposes, the transfer function matrix $G(\xi)$ is considered to be in the form of the ratio of a matrix numerator polynomial $B(\xi)$ and an nth-order monic scalar denominator polynomial $a(\xi)$, i.e..

$$G(\xi) = \frac{B(\xi)}{a(\xi)} \tag{3}$$

$$B(\xi) = B_0 + B_1 \xi^{-1} + \ldots + B_n \xi^{-n} \tag{4}$$

$$a(\xi) = 1 + a_1 \xi^{-1} + \ldots + a_n \xi^{-n} \tag{5}$$

where $B_k \in \Re^{n_y \times n_u}, k = 0,\ldots,n$. The model order n should be chosen as an upper bound on the true plant order so that the optimization problem is *overparam etrized*.

Several algorithms are presently available for solving (1). These methods tend to be of two types, fixed-point iterations [1][7][13][16] [19][20], or fixed-point iterations combined with modified gradient methods [4][17] [20].

A simple and effective, (but approximate) algorithm for minimizing (1) is found in the work of Sanathanan and Koerner [16]. The algorithm was introduced for SISO plants in the Laplace s operator, but is easily extended to the multivariable case in the arbitrary operator ξ.

SK Iteration:

$$a^{k+1}, B^{k+1} = \arg\min_{a,B} \sum_{i=1}^{m} \mathrm{w}^2(\omega_i) \left\| \frac{1}{a^k(\xi(\omega_i))} \Big(G(\omega_i) a(\xi(\omega_i)) - B(\xi(\omega_i)) \Big) \right\|_f^2 \tag{6}$$

with initial condition $a^0 = 1, B^0 = 0$. With a^k fixed at each iteration, the cost function in (6) is quadratic. Hence, the SK iteration is implemented as a sequence of linear least squares problems. As proposed in [4], an effective approach is to use the SK iteration as an initializer for a Gauss-Newtion (GN) iteration. Details on the SK and GN iterations with choices of polynomial basis for curve fitting in the z, δ and s operators are given in [4]. Also, sparse matrix methods are introduced in [4] which make use of the special strut.ture of the matrices arising in the SK and GN iterations for the multivariable case.

Step 2. *Choose any $N \geq 2n + 2$, and solve for Markov parameters H_i, $i = 0, ..., N$.* Given G, one can divide $a(\xi)$ into $B(\xi)$ to give the Markov parameter sequence $\{H_i\}$,

$$\frac{B(\xi)}{a(\xi)} = \sum_{i=0}^{\infty} H_i \xi^{-i} \tag{7}$$

which gives upon cross-multiplying,

$$B_0 + B_1\xi^{-1} + ... + B_n\xi^{-n} = (1 + a_1\xi^{-1} + ... + a_n\xi^{-n}) \sum_{i=0}^{\infty} H_i \xi^{-i} \tag{8}$$

Equating coefficients of the first N powers of ξ^{-1} in (8) gives the following system of linear equations,

$$\begin{bmatrix} I & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ a_1 I & I & \ddots & & & & \\ & a_2 I & I & \ddots & & & \\ a_n I & \ddots & a_1 I & I & \ddots & & \\ 0 & \ddots & & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & & \ddots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & a_n I & \cdots & a_1 & I \end{bmatrix} \begin{bmatrix} H_0 \\ H_1 \\ \vdots \\ \vdots \\ H_n \\ H_{n+1} \\ \vdots \\ H_N \end{bmatrix} = \begin{bmatrix} B_0 \\ B_1 \\ \vdots \\ \vdots \\ B_n \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{9}$$

Given the estimated polynomial $a(\xi)$ and polynomial matrix $B(\xi)$, the multivariable Markov parameters $H_i \in \Re^{n_y \times n_u}$, $i = 0, \ldots, N$ can be calculated by solving the above system of equations. Since the matrix to be inverted is lower triangular with ones on the diagonal, it is always invertible and a solution always exists. Furthermore, since this system of equations is block triangular it can be solved by backsubstitution giving rise to the following recursive formula,

$$H_0 = B_0 \tag{10a}$$

$$H_k = B_k - \sum_{j=1}^{k} a_j H_{k-j}; \quad k = 1, \ldots, n \tag{10b}$$

$$H_k = - \sum_{j=1}^{n} a_j H_{k-j}; \qquad k = n+1, \ldots, N \tag{10c}$$

Step 3. *Choose any r and s such that $r + s \leq N - 2$ and $min(r, s) \geq n$, and form the Hankel type matrices $H(0)$, $H(1) \in \Re^{n_y(r+2) \times n_u(s+2)}$ where,*

$$H(0) = \begin{bmatrix} H_0 & H_1 & \cdots & H_{s+1} \\ H_1 & H_2 & \cdots & H_{s+2} \\ \vdots & \vdots & \ddots & \vdots \\ H_{r+1} & H_{r+2} & \cdots & H_{r+s+1} \end{bmatrix} \tag{11}$$

$$H(1) = \begin{bmatrix} H_1 & H_2 & \cdots & H_{s+2} \\ H_2 & H_3 & \cdots & H_{s+3} \\ \vdots & \vdots & \ddots & \vdots \\ H_{r+2} & H_{r+3} & \cdots & H_{r+s+2} \end{bmatrix} \tag{12}$$

Step 4. *Compute a balanced state-space realization using the ERA algorithm, i. e.,*

4.a Compute the SVD of $H(0)$ to give,

$$H(0) = U \Sigma V^T \tag{13}$$

where $\mu = min(n_y(r+2), n_u(s+2)) + 2$, $U \in \Re^{n_y(r+2) \times \mu}$, $V \in \Re^{n_u(s+2) \times \mu}$, $\Sigma = diag\{\sigma_1, \ldots, \sigma_\mu\}$, and the singular values are ordered by size, $\sigma_i \geq \sigma_{i+1}$, $i = 1, \ldots, \mu - 1$

4. b Plot the Hankel singular values σ_i to visualize trade-off between model order and identification accuracy, and truncate to keep only q singular values.

4. c Form q-th order state-space realization in the operator ξ as,

$$\xi x = A_q x + B_p u \tag{14}$$

$$y = C_q x + D u \tag{15}$$

where,

$$A_q = \Sigma_q^{-1/2} U_q^T H(1) V_q \Sigma_q^{-1/2} \tag{16.a}$$

$$B_q = \Sigma_q^{1/2} V_q^T E_u \tag{16.b}$$

$$C_q = E_y^T U_q \Sigma_q^{1/2} \tag{16.c}$$

$$D = H(0) \tag{16.d}$$

$$E_y^T = [I_{n_y \times n_y} \;\; O] \in \mathcal{R}^{n_y \times n_y(r+2)} \tag{17.a}$$

$$E_u^T = [I_{n_u \times n_u} \;\; O] \in \mathcal{R}^{n_u \times n_u(s+2)} \tag{17.b}$$

$$\Sigma_q = diag\{\sigma_1, \ldots, \sigma_q\}$$

U_q - submatrix formed from first q columns of U

V_q - submatrix formed from first q columns of V

■

3. OVERPARAMETRIZATION IN COMPLEX CURVE FITTING

A key property of the SSFD algorithm is that the model order in the complex curve fitting step can be overspecified. This is because near pole-zero cancellations in the curve fit form weakly uncontrollable and/or unobservable subspaces in the state-space realization and are systematically eliminated based on Hankel singular values.

The effects of overparametrization in general problems of complex curve fitting will be analyzed in this section. We will need the following lemma,

Lemma 1 *A monic polynomial*

$$c(\xi) = 1 + c_1\xi^{-1} + \ldots + c_\ell \xi^{-\ell} \tag{18}$$

is stable if and only if its coefficient vector $c_v = [c_1, \ldots, c_\ell]^T$ can be written as,

$$c_v = -R_{22}^{-1} r_{21} \tag{19}$$

where $R_{22} \in \mathcal{R}^{\ell \times \ell}$ and $r_{21} \in \mathcal{R}^{\ell}$ are determined from a matrix partition of a symmetric positive definite Toeplitz matrix $R \in \mathcal{R}^{(\ell+1)\times(\ell+1)}$, i.e.,

$$R = \begin{bmatrix} r_{11} & | & r_{21}^T \\ -- & | & ---- \\ r_{21} & | & R_{22} \end{bmatrix} \tag{20}$$

Proof: It is shown in Vieira and Kailath ([18], see Lemma on page 219 and eqn (13)) that a polynomial $c(\xi)$ has all of its roots inside the unit circle if and only if,

$$R \begin{bmatrix} c_0 \\ c_v \end{bmatrix} = \begin{bmatrix} \beta \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{21}$$

where, $R \in \mathcal{R}^{(\ell+1)\times(\ell+1)}$ is a symmetric positive definite Toeplitz matrix, and the scalar $\beta \neq 0$ can be chosen arbitrarily since it scales $c(\xi)$ and will not affect the polynomial's roots. Substituting the partitioned structure of R (20) into (21) gives,

$$r_{11} c_0 + r_{21}^T c_v = \beta \tag{22}$$

$$r_{21} c_0 + R_{22} c_v = 0 \tag{23}$$

Solving for c_0 and c_v simultaneously in (22)(23) and letting $\beta = r_{11} - r_{21}^T R_{22}^{-1} r_{21} = det(R)/det(R_{22}) > 0$ gives $c_0 = 1$ (hence $c(\xi)$ is monic) and $c_v = -R_{22}^{-1} r_{21}$, as desired. ∎

The main theoretical result of this paper is given next.

Theorem 1 *Let the plant be given by the following rational transfer function in the complex variable ξ,*

$$G(\xi) = \frac{b(\xi)}{a(\xi)} \tag{24}$$

$$a(\xi) = 1 + a_1 \xi^{-1} + \ldots + a_n \xi^{-n} \tag{25.a}$$

$$b(\xi) = b_0 + b_1 \xi^{-1} + \ldots + b_n \xi^{-n} \tag{25.b}$$

where polynomials $a(\xi)$ and $b(\xi)$ are coprime. Assume that noiseless data $G(\xi_i)$ is specified at N distinct values of ξ_i, $i = 1, \ldots, N$, none of which coincide with roots of $a(\xi)$ or $b(\xi)$; and let overparametrized polynomials $\bar{a}$ and $\bar{b}$ of order $N \geq \bar{n} > n$ satisfy the data, i.e.,

$$0 = -G(\xi_i)\bar{a}(\xi_i) + \bar{b}(\xi_i), \quad i = 1, \ldots, N \tag{26}$$

$$\bar{a}(\xi) = 1 + \bar{a}_1 \xi^{-1} + \ldots + \bar{a}_{\bar{n}} \xi^{-\bar{n}} \tag{27a}$$

$$\bar{b}(\xi) = \bar{b}_0 + \bar{b}_1 \xi^{-1} + \ldots + \bar{b}_{\bar{n}} \xi^{-\bar{n}} \tag{27b}$$

Then, the following properties hold:

$$\begin{array}{ll} \textbf{(i)} & a(\xi)\bar{b}(\xi) = b(\xi)\bar{a}(\xi) \\ \textbf{(ii)} & \bar{a}(\xi) = a(\xi)c(\xi) \\ \textbf{(iii)} & \bar{b}(\xi) = b(\xi)c(\xi) \end{array}$$

where $c(\xi)$ is a monic polynomial of order $\ell = (\bar{n} - n)$, i.e.,

$$c(\xi) = 1 + c_1 \xi^{-1} + \ldots + c_\ell \xi^{-\ell} \tag{28}$$

Furthermore, let a unique solution to (26) be obtained as,

$$\bar{\theta} = H^{\dagger} y \tag{29}$$

where $\dagger$ denotes the Moore-Penrose inverse, and (26) has been rearranged into matrix form as follows,

$$H\bar{\theta} = y \tag{30}$$

$$\bar{\theta} = [\bar{\theta}_a^T, \bar{\theta}_b^T]^T \tag{31a}$$

$$\bar{\theta}_a = [\bar{a}_1, ..., \bar{a}_{\bar{n}}]^T; \quad \bar{\theta}_b = [\bar{b}_0, \bar{b}_1, ..., \bar{b}_{\bar{n}}]^T \tag{31b}$$

$$H = \begin{bmatrix} \Re\{\mathcal{H}\} \\ \Im\{\mathcal{H}\} \end{bmatrix}; \quad y = \begin{bmatrix} \Re\{g\} \\ \Im\{g\} \end{bmatrix} \tag{32}$$

$$\mathcal{H} = \left[\begin{array}{ccc|ccc} -G(\xi_1)\xi_1^{-1} & \dots & -G(\xi_1)\xi_1^{-\bar{n}} & \xi_1^{-} & \dots & \xi_1^{-\bar{n}} \\ \vdots & & \vdots & \vdots & & \vdots \\ -G(\xi_N)\xi_N^{-1} & \dots & -G(\xi_N)\xi_N^{-\bar{n}} & \xi_N^{-} & \dots & \xi_N^{-\bar{n}} \end{array}\right] \tag{33}$$

$$g = [G(\xi_1), ..., G(\xi_N)]^T \tag{34}$$

Then,

(iv) *all ℓ roots of $c(\xi)$ lie strictly inside the unit circle.*

Proof: Substituting G from (24) into (26), multiplying both sides by $a(\xi)$ and evaluating on ξ_i for $i = 1, ..., \bar{n}$ gives the system of equations,

$$a(\xi_i)\bar{b}(\xi_i) = b(\xi_i)\bar{a}(\xi_i) \quad i = 1, ..., \bar{n} \tag{35}$$

Result (i) follows by writing (35) in matrix form, using the fact that the resulting Vandermonde matrix is nonsingular when formed from powers of the distinct values ξ_i [11].

Continuing, since a and b are coprime, there exist polynomials $u(\xi)$ and $v(\xi)$ such that,

$$1 = u(\xi)a(\xi) + v(\xi)b(\xi) \tag{36}$$

Multiplying both sides of (36) by $\bar{a}(\xi)$ and using result (i) gives,

$$\bar{a}(\xi) = \bar{a}(\xi)a(\xi)u(\xi) + \bar{a}(\xi)b(\xi)v(\xi)$$

$$= \bar{a}(\xi)a(\xi)u(\xi) + \bar{b}(\xi)a(\xi)v(\xi) = \Big(\bar{a}(\xi)u(\xi) + \bar{b}(\xi)v(\xi)\Big)a(\xi) \tag{37}$$

Letting $c(\xi) = \bar{a}(\xi)u(\xi) + \bar{b}(\xi)v(\xi)$ in (37) proves result (ii). Likewise, result (iii) is proved by multiplying both sides of (36) by $\bar{b}(\xi)$ and using (ii) to give,

$$\bar{b}(\xi) = \bar{b}(\xi)a(\xi)u(\xi) + \bar{b}(\xi)b(\xi)v(\xi)$$

$$= \bar{a}(\xi)b(\xi)u(\xi) + \bar{b}(\xi)b(\xi)v(\xi) = \Big(\bar{a}(\xi)u(\xi) + \bar{b}(\xi)v(\xi)\Big)b(\xi) \tag{38}$$

Result (iv) is proved next. It follows by definition of the Moore 1 'enrose inverse [2] that the solution in ('29) is equivalent to the solution of the optimization problem,

$$\min_{\bar{\theta}} ||\bar{\theta}||^2 \tag{39}$$

subject to,

$$H\bar{\theta} = y \tag{40}$$

However, in light of results (ii) and (iii), this is also equivalent to the following problem,

$$\min_{c_v} \left(1 + ||\bar{\theta}||^2\right) \tag{41}$$

where $c_v = [c_1, \ldots, c_\ell]^T$, and without 10ss of generality a '(1" was added to the cost (this does not affect the minimization). It is noted that the polynomial identities in (ii) and (iii) can be put into matrix form as,

$$\begin{bmatrix} 1 \\ \bar{\theta} \end{bmatrix} = \begin{bmatrix} \mathcal{A} \\ \mathcal{B} \end{bmatrix} \begin{bmatrix} 1 \\ c_v \end{bmatrix} \tag{42}$$

where $\mathcal{A} = [\ell_a|\bar{\mathcal{A}}]$ is Toeplitz with first column $\ell_a = [1, a_1, \ldots, a_n, 0, \ldots, 0]^T$ and $\mathcal{B} = [\ell_b|\bar{\mathcal{B}}]$ is Toeplitz with first column $\ell_b = [b_0, b_1, \ldots, b_n, 0, \ldots, 0]^T$. The constrained optimization problem (41) is now completely in matrix notation and can be solved to give the closed-form solution,

$$c_v = -R_{22}^{-1} r_{21} \tag{43}$$

where the matrix,

$$R = \mathcal{A}^T\mathcal{A} + \mathcal{B}^T\mathcal{B} \tag{44}$$

is partitioned as in Lemma A.1 with $r_{11} = \ell_a^T\ell_a + \ell_b^T\ell_b$; $r_{21} = \bar{\mathcal{A}}\ell_a + \bar{\mathcal{B}}\ell_b$; and $R_{22} = \bar{\mathcal{A}}^T\bar{\mathcal{A}} + \bar{\mathcal{B}}^T\bar{\mathcal{B}}$. From the symmetric positive definite Toeplitz structure of R in (44) it follows from Lemma A.1 that the roots of

$$c(\xi) = 1 + c_1\xi^{-1} + \ldots + c_\ell\xi^{-\ell} \tag{45}$$

lie inside the unit circle which is the desired result (iv). ■

4. DISCUSSION

Theorem 1 is important because it indicates (assuming noiseless data and infinite precision arithmetic) that the true plant dynamics will be a subset of overparametrized plant dynamics and the extraneous poles and zeros come in as exact pole-zero cancellations. Finally, it specifies that the extraneous dynamics introduced from overparametrization will be inside the unit circle (assuming that the pseudoinverse is used to ensure a unique solution to the overparametrized problem, and that the polynomials are fitted monic in their highest power). This means that *a complex operator overparametrizes stably if and only if its stability region fully encompasses the unit disk.* We have the following immediate results,

1) The shift Operator $\xi = z$ overparametrizes stably since its stability region is identical to the unit disk. Note that the polynomials (27) in Theorem 1 are defined in causal form, and hence the choice $\xi = z^{-1}$ corresponds to a noncausal form, which will not overparametrize stably.

2) The delta operator $\xi = \delta = (z - 1)/T$ dots not overparametrize stably since the delta-rule stability disk only partially encompasses the unit disk.

3) The Laplace s operator does not overparametrize stably because its stability region (i.e., the left half plane) only encompasses half of the unit disk.

4) The delta operator can be modified to $\bar{\delta} = \beta + \frac{(z-1)}{\alpha}$ which overparametrizes stably for any $0 < \alpha \leq 1$ and $1 \leq \beta \leq (2 - alpha)/alpha$. Note that letting $\alpha = 1$, $\beta = 1$ recovers the shift operator, while the choices $\alpha = T$ and $\beta = 1$ gives the new operator,

$$\bar{\delta} = \frac{(z - 1 + T)}{T}$$

which for small T is only a slight modification of the original delta-rule definition

5) The Laplace s operator can be modified to $\bar{s} = s + \beta$ which overparametrizes stably for any $\beta \geq 1$.

Maintaining stability in the face of overparametrization ensures that a stable plant will be estimated as a stable plant (this is important for robust, control analysis multivariable Nyquist Theorem), and furthermore that no artificial nonminimum phase zeros will be introduced inadvertently in the identification effort (this is important for any approach to control). Furthermore, powers of z form a natural orthogonal basis when evaluated on the unit circle which is ideal for estimation purposes. Finally, using z, it can be shown that the restriction of the poles to the unit disk ensures that the condition number of the matrix in (9) (required to be inverted in the SSFD algorithm) is relatively small. These three properties indicate the reason for the success achieved in [4] [6] and many other approaches in the literature which use powers of z as a polynomial basis.

Unfortunately, none of these nice properties carry over to the delta-mlc and Laplace operators. While some modifications for overcoming these difficulties are given in the above discussion (for stable overparametrization), and in [4] (for modifying the power basis to an orthogonal Chebychev basis), further efforts are required to develop a completely satisfying approach to curve fitting in these operators.

5. EXPERIMENTAL CASE STUDY

This example demonstrates the SSFD algorithm on experimental data taken from the J 1'1, Advanced Reconfigurable Control Testbed. A 4-input, 3-output transfer function is considered, where each actuator is an active stint, and each sensor is an accelerometer. The frequency response data is obtained using a 512 Schroeder phased sum-of-sinusoids input design at a sampling rate of 200 Hertz (background on the design of Schroeder-phased inputs, and their use in unbiased estimation can be found in [3]). The magnitude response is shown as the dashed line in Fig. 2 (phase is available but not shown).

Let the overparametrized model order be $n = 60$ in Step 1 of the SSFD algorithm, and let $w(\omega_i) = 1$, (i.e., a uniform weighting). Since there are 12 numerator polynomials and 1 denominator polynomial, this requires the simultaneous estimation of 780 parameters. The SK algorithm is iterated 12 times, using the the sparse matrix SVD algorithm developed in [4]. The sparse matrix SVD algorithm was indespensible for this problem, reducing RAM requirements by better than an order of magnitude (from approximately 60 Megabytes to 6 Megabytes) and reducing computation time two orders of magnitude (from approximately 33 hours to 20 minutes).

Steps 2, 3, and 4 (the realization portion) of the SSFD algorithm are computed using $r = 80$, $s = 240$. The singular values are plotted in Fig. 1. It is seen that there is a sharp drop off at 180 states since there is an *exact* state-space realization of this size (i. e., $180 = \min(n_y, n_u) * 60$). However, for demonstration purposes, a model order of 100 is chosen (i.e., $q = 100$). This is also reasonable since the error from the singular value plot is seen to have dropped approximately 3 orders of magnitude at this point.

A magnitude plot of the state-space model realized from Step 4 of the algorithm is shown in Fig. 2 (solid line) superimposed on the response data (dashed line). The state space model is stable, and is seen to match the data well over a considerable bandwidth.

6. CONCLUSIONS

The SSFD algorithm has been extended to estimating state-space models in arbitrary operators. In order to support this extension, a result was proved characterizing the extraneous dynamics introduced by overparametrization. The main result is surprisingly simple, indicating that *the extraneous dynamics are always inside the unit circle regardless of the operator being used* (this result assumes no noise, infinite precision arithmetic, use of the Moore-Penrose pseudoinverse, and that the polynomials are fitted monic in their highest power). This result is also important to all areas of identification and estimation since it indicates that the shift operator is well behaved with respect to overparametrization while the delta-rule and Laplace operators are not. Appropriate modifications of these latter operators was given to ensure that they overparametrize stably. However, additional efforts are required to obtain results comparable to the good performance experienced using the shift operator formulation of the SSFD algorithm.

The SSFD algorithm (in the shift operator) was tested on experimental data set demonstrating the successful identification of a multivariable (4-input/3-output) 100 state model over a bandwidth of 100 Hertz. In this case, the SSFD algorithm was used in conjunction with frequency data acquired using the multisinusoidal input designs in [3]. The general results are encouraging, and indicate that the approach would be useful in such areas as adaptive optics, flexible structures, helicopter/rotocraft testing, high performance tracking) or any other applications requiring the accurate identification of high-order multivariable systems over wide bandwidths.

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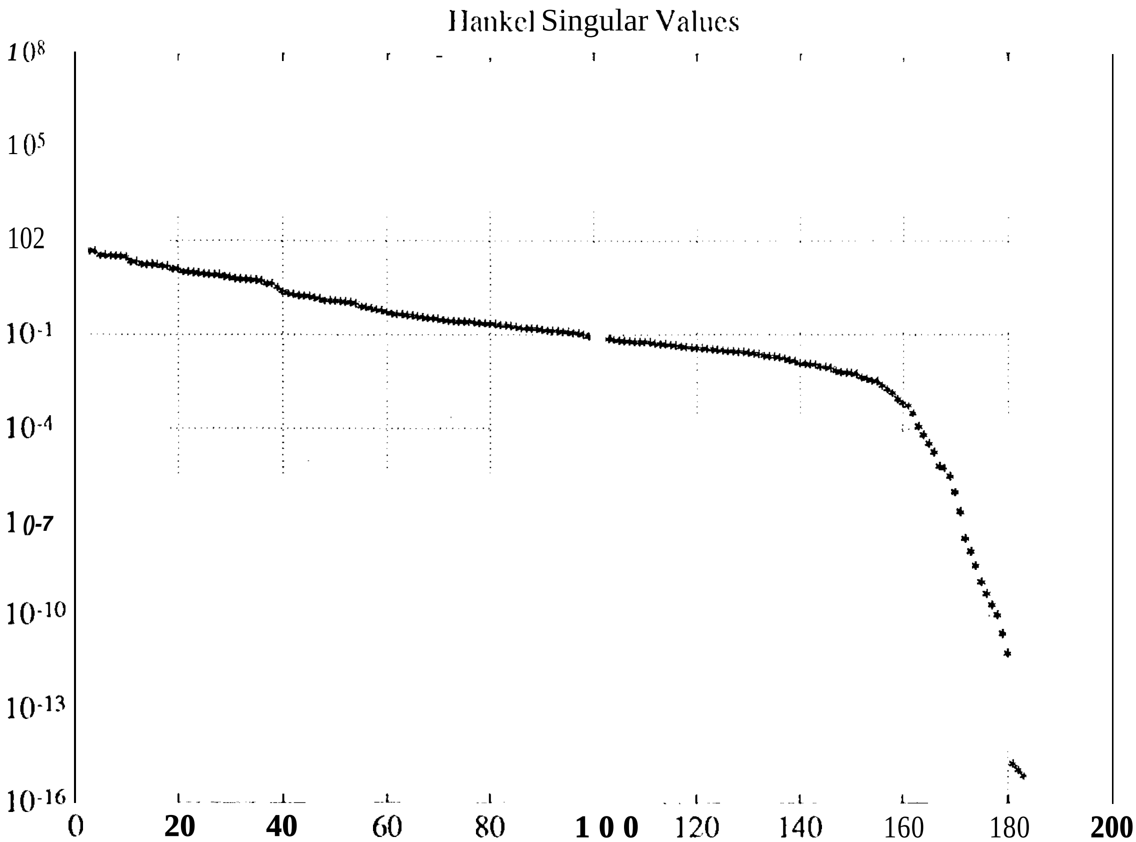

Fig. 1 Hankel singular values

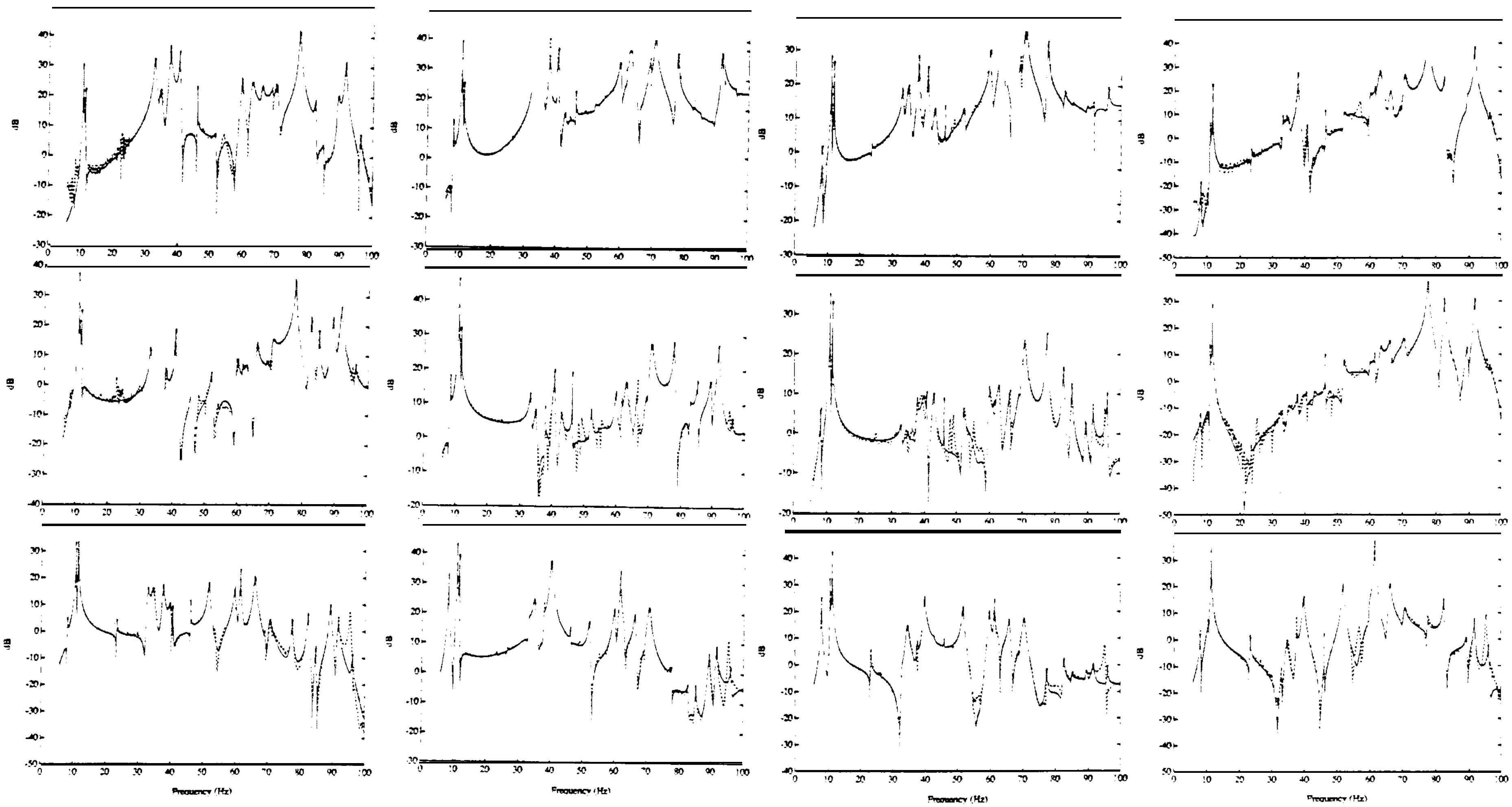

Fig. 2 Identification results using SSFD algorithm: Identified 100 state 4-input/3-output state-space model (solid); Raw experimental data (dashed)